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Friedmann, Robertson-Walker (FRW) Models in Cosmology

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Abstract

Friedmann, Robertson-Walker (FRW) models are established on the basis of the assumption that the universe is homogeneous and isotropic in all epochs. Even though the universe is clearly inhomogeneous at the local scales of stars and cluster of stars, it is generally argued that an overall homogeneity will be achieved only at a large enough scale of about 14 billion light years. According to the FRW models, the universe has an encompassing space-time singularity at a finite time in the past. This curvature singularity is called the big bang. FRW singularity must be interpreted as the catastrophic event from which the entire universe emerged, where all the known laws of physics and mathematics breakdown in such a way that we cannot know what was happened during and before the big bang singularity. In these models the three-space is flat and are of positive and negative constant curvature; which incorporate the closed and open FRW models respectively. In this paper an attempt has been made to describe the FRW models with easier mathematical calculations, physical interpretations and diagrams where necessary.

Keywords: Big bang, FRW models, Homogeneous and isotropic universe, Hubble constant.

1 Introduction

From the ancient period human had curiosity about the universe. The astronomers then believed that the Earth is in the centre of the universe and all the celestial bodies rotate around the Earth. Nicolaus Copernicus first expressed that the Sun is at the center of the solar system with the Earth in orbit around the Sun. Newton also believed, according to his theory, that the universe is static and infinite.

Einstein expressed that the geometry of space is described by the Riemannian geometry of four dimensions: three spatial and one temporal. In the Einstein field equations he stated that the universe is static. Einstein introduced a cosmological constant $\Lambda(\approx 0)$ for static universe solutions as;

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda\, g_{\mu\nu} = -\frac{8\pi\, G}{c^4} T_{\mu\nu}.$$

In the 1930s Edwin Hubble observed that the galaxies are receding from each other by the observation of red shift of the light rays emitting from the stars. He expressed that the universe is not static and it is expanding. Before his observation the Russian mathematician A. A. Friedmann (1888–1925) established his models in 1922 and for the first time he expressed that the universe is expanding. In (t, r, θ, ϕ) coordinates the Robertson-Walker line element is given by;

$$ds^2 = -dt^2 + S^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

where $S(t)$ is the scale factor and k is a constant which denotes the spatial curvature of the three-space and could be normalized to the values +1, 0, –1. When k = 0 the three-space is flat and the model is called Einstein de-Sitter static model, when k = +1 and k = –1 the three-space are of positive and negative constant curvature; these incorporate the closed and open Friedmann models respectively.

In the 1960s Stephen W. Hawking and Roger Penrose [6] discovered the singularities in the FRW model. At the epoch of big bang, the entire three-surface shrinks to zero volume and the densities and curvatures grow to infinity. Hence by FRW models, the universe has an encompassing space-time a singularity during the big bang at a finite time in the past.

2 Earlier Concepts about the Universe

In the earliest cosmologies human placed themselves in the commanding position at the centre of the universe. Before Nicolaus Copernicus (1473–1543) period it is believed that the Earth is in the centre of the universe and other stars, planets and satellites move around the Earth. The Aristotelian model of the solar system in the Middle Ages placed the Earth at the center of the solar system, a unique place since it appears that everything revolved around the Earth. Nicolaus Copernicus first demonstrated that this view was incorrect and that the Sun was at the center of the solar system with the Earth in orbit around the Sun. Bondi [1] named this principle as Copernican principle. Since the time of Copernicus we admit that humans are not privileged observers of the universe.

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Rowan-Robinson [15] emphasizes the Copernican principle as the threshold test for modern thought, asserting that: “It is evident that in the post-Copernican era of human history, no well-informed and rational person can imagine that the Earth occupies a unique position in the universe.”

3 Modern Concepts about the Universe

Human has many questions about the universe always. How is the universe? What is its structure? Is it infinite? Or, has it a limit? Is it eternal? Or, did it have a beginning? And it will have an end? etc. The first scientific theory which was able to give answers properly, and that gave rise to a cosmology, was the Newton’s Mechanics. He supposed a space with a Euclidian geometry and an absolute time for the entire universe. Newton also believed, according to his theory, that the universe was static and infinite. He expressed that the matter in the universe was in balance because its distribution was uniform and infinite, in such a way that each of the stars is balanced with its neighbors, by the entire universe [11]. He postulated the idea of an infinite universe in order to avoid the stars falling into a hypothetical centre of mass of a finite space.

Einstein expressed that the geometry of space is described by the Riemannian geometry of four dimensions: three spatial and one temporal. He proposed his field equation that relates the distribution of matter and the energy with the curvature of this space-time. He also expressed that the universe is static. He introduced a cosmological constant $(\Lambda \approx 0)$ to allow for a static Universe. When Edwin Hubble discovered his fundamental law describing the universal expansion, the prerequisites of infiniteness and Λ was not needed anymore. In spite of that, Λ still appears in the fundamental equations describing the standard cosmological model [4].

We are the inhabitants of the 21st century. But we are not yet able to make cosmological models without admixture of ideology. Since the time of Copernicus we have been steadily demoted to a medium sized star on the outer edge of a fairly average galaxy, which itself simply one of a local group of galaxies. Indeed we are now so democratic that we would not claim that our position in space is especially distinguished in any way [6].

A reasonable interpretation of this somewhat blurred principle is to understand it as implying that, when viewed on a suitable scale, the universe is approximately spatially homogeneous. By spatially homogeneous, we mean there is a group of isometries which acts freely on the manifold *M*, and whose surfaces of transitivity are spacelike three-surfaces. In other words, any point on one of these surfaces is equivalent to any other point on the same surface. Here isometries mean that a tensor $g_{\mu\nu}(x)$ is form invariant under a transformation from x^{μ} to x'^{μ} i.e., $g'_{\mu\nu}(x) = g_{\mu\nu}(x)$ for all *x*.

The homogeneity in space means that the universe is roughly the same at all spatial points and that the matter is uniformly distributed all over the space. This is an assumption difficult to check. Even though the universe is clearly inhomogeneous at the local scales of stars and cluster of stars, it is generally argued that an overall homogeneity will be achieved only at a large enough scale of about 14 billion light

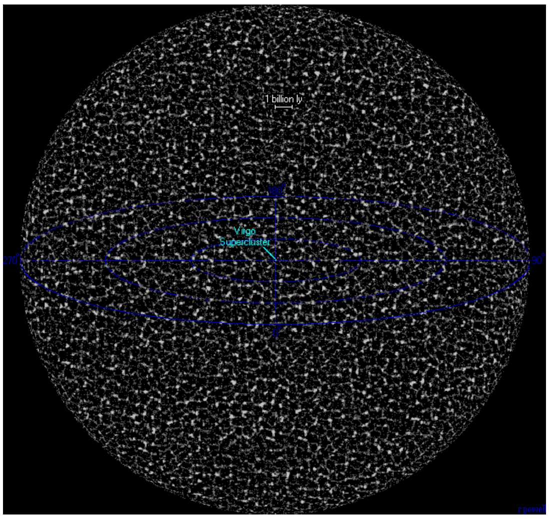

Figure 1: Atlas of the visible universe (14 billion light years of the sun). To this scale the universe is fairly uniform (Atlas of the universe, from Richard Powell).

years (because the universe is about 14 billion years old) or so (undetermined scale), in a statistical sense only (figure 1). The light from more distant objects simply has not had time to reach us. For this reason everybody in the universe will find themselves at the middle of their own visible universe. The precise scale of the universe is complicated by the fact that the universe is expanding. Galaxies we see near the edge of the visible universe emitted their light when they were much closer to us, and they will now be much further away.

It is possible to have observational tests on the assumption of isotropy, that is, the universe must be the same in all directions. One could check the distribution of galaxies in the different directions together with their apparent magnitudes and red-shifts, and also the distribution of radio sources similarly. Such observations are again interpreted frequently as providing an evidence for isotropic distribution of matter in the universe from our vantage point. Again, the observed microwave background radiation appears to be isotropic to a high degree of approximation in all directions. Then, if this radiation is of cosmic origin, it would imply that the perturbations from overall isotropy should be very large on our past light cone to which all our observations are confined.

If we assume the isotropy of the universe and combine it with the assumption of isotropy of the cosmological principle, which is generally given by the statement that we do not occupy any special position in the universe, then the assumption of isotropy can be extrapolated to hold at all points of the universe. In such a case, the universe is spherically symmetric around all its points as opposed to the situation of the asymptotically flat space-times such as the Schwarzschild [12], where there is a spherical symmetry around the centre. In terms of the mathematical model of space-time, the homogeneity assumption could be interpreted as saying that the space-time is a stockpile of spacelike hypersurfaces each defining a constant value of time and given any two points *p* and *q* on one of these hypersurfaces, there is an isometry of the metric tensor *g* which takes from a point *p* to another point *q*.

4 Topological properties of the Universe

Topologically manifold $M = \Sigma \times \Re$, where Σ is a three-dimensional spacelike hypersurface and also the space-time is globally hyperbolic in the sense that all the non-spacelike curves in *M* must intersect Σ once and only once either in the past or in the future. Here $\Re$ stands for time which is called the *cosmic time* or *cosmological time* which is valid globally and is defined as follows: Introduce a series of non-interacting spacelike hypersurfaces, that is, surfaces any two points of which can be connected to each other by a curve lying entirely in hypersurface which is spacelike everywhere. We assume that all the galaxies lie on such a hypersurface in such a manner that the surface of simultaneity of the local Lorentz frame of any galaxy coincides locally with the hypersurface (figure 2).

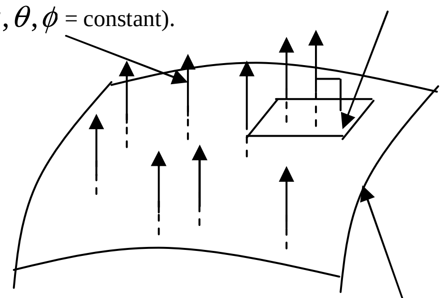

Figure 2: Representation of a typical spacelike hypersurface on which galaxies are assumed to lie.

In other words, all the local Lorentz frames of the galaxies mesh together to form the hypersurface. So that the 4-velocity $u^i, i = 1,2,3,4$; of a galaxy is orthogonal to the hypersurface. This series of hypersurfaces can be labeled by a parameter which may be taken as the proper time of any galaxy, that is, time as measured by a clock stationary in the galaxy, which is a cosmic time. As we shall see, this defines a universal time, so that, a particular time means a given spacelike hypersurface on this series of hypersurfaces.

Here along the worldliness, the galaxies, or even the clusters of galaxies are to be taken as points while modeling the universe and writing its line element depicting the global geometry.

According to the Weyl's postulate [16] the world-lines of galaxies are a bundle or congruence of geodesics in space-time diverging from a point in the (finite or infinite distance) past or converging to such a point the future (figure 3) where the space coordinates of the galaxy remain constant and time coordinate is proper time along such a geodesic world line [9]. These geodesics are non-intersect except possibly at a singular point in the past or future or both. There is one and only one such geodesic passing through each regular (non-regular) space-time point. This assumption is satisfied to a high degree of accuracy in the actual universe.

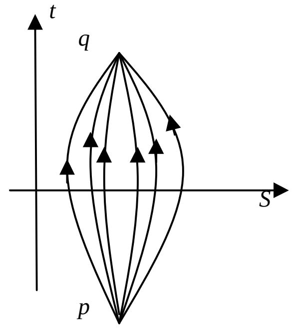

Figure 3: Congruence of geodesics in space-time diverging from a point *p* in the (finite or infinite distance) past or converging to such a point *q* the future.

We assume that the bundle of geodesics satisfying Weyl's postulate possesses a set of spacelike hypersurface orthogonal to them choose a parameter such that each of these hypersurfaces corresponds to $t = \text{constant}$. The parameter *t* can be chosen to measure the proper time along a geodesic. Now introduce spatial coordinates (x^1, x^2, x^3) which are constant along any geodesic. Thus for each galaxy the coordinates (x^1, x^2, x^3) are constant. Under these circumstances the metric can be written as follows:

$$ds^2 = c^2 dt^2 - h_{ij} dx^i dx^j \quad (i, j = 1,2,3) \tag{1}$$

where the h_{ij} are functions of (t, x^1, x^2, x^3). Let the world-line of a galaxy be given by $x^\mu(\tau)$, where τ is the proper time along the galaxy. Then according to our assumptions $x^\mu(\tau)$ is given as follows:

$$x^0 = c\tau, \quad x^1, x^2 x^3 = \text{constant} \ . \tag{2}$$

Hence from (2) we get;

$$dx^i = 0, \quad i = 1,2,3. \tag{3}$$

From (1) and (3) we get;

$ds = cd\tau = cdt \Rightarrow \tau = t$ i.e., along the galaxy proper time τ is coordinate time *t*. Clearly a vector along the world-line is given by $A^\mu = (cdt, 0,0,0)$ and the vector $B^\mu = (0, dx^1, dx^2, dx^3)$ lying $\sum, t = \text{constant}$ are orthogonal i.e.,

$$g_{\mu\nu} A^\mu B^\nu = 0 \ . \tag{4}$$

5 Friedmann, Robertson–Walker (FRW) Models

The FRW models play an important role in Cosmology. These models are established on the basis of the homogeneity and isotropy of the universe as described above. These models are among the most popular backgrounds in gravitational physics. There are several reasons for the popularity of FRW models. The current observations give a strong motivation for the adoption of the cosmological principle stating that at large scales the universe is homogeneous and isotropic and, hence, its large-scale structure is well described by the FRW metric. The FRW geometries are related to the high symmetry of these backgrounds. Due this symmetry numerous physical problems are exactly solvable and a better understanding of physical effects in FRW models could serve as a handle to deal with more complicated geometries.

Einstein introduced a cosmological constant $\Lambda(\approx 0)$ for static universe solutions as;

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda \, g_{\mu\nu} = -\frac{8\pi\, G}{c^4} T_{\mu\nu}. \tag{5}$$

Einstein's field equation (5) for $\Lambda = 0$ can be written as;

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi\, G}{c^4} T_{\mu\nu}. \tag{6}$$

where $G = 6.673\times10^{-11}$ is the gravitational constant and $c = 10^8$ *m/s* is the velocity of light but in relativistic unit $G = c = 1$. Hence in relativistic units (6) becomes;

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi T_{\mu\nu}\ . \tag{7}$$

It is clear that divergence of both sides of (6) and (7) is zero. For empty space $T_{\mu\nu} = 0$ then $R_{\mu\nu} = \Lambda \, g_{\mu\nu}$, then;

$$R_{\mu\nu} = 0 \text{ for } \Lambda = 0 \tag{8}$$

which is Einstein's law of gravitation for empty space.

In (t, r, θ, ϕ) coordinates the Robertson-Walker line element is given by;

$$ds^2 = -dt^2 + S^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta \, d\phi^2\right)\right] \tag{9}$$

where $S(t)$ is the scale factor and k is a constant which denotes the spatial curvature of the three-space and could be normalized to the values +1, 0, –1. When $k = 0$ the three-space is flat and (9) is called Einstein de-Sitter static model, when $k = +1$ and $k = -1$ the three-space are of positive and negative constant curvature; these incorporate the closed and open Friedmann models respectively (figure 4). Here coordinate t is timelike and other coordinates r, θ, ϕ are spacelike, θ and ϕ are the corresponding angular coordinates in the co-moving frame.

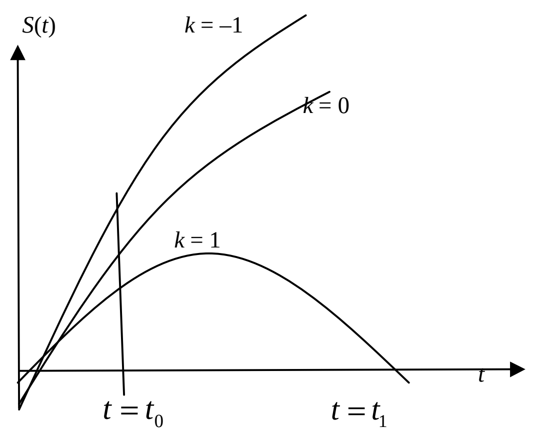

Figure 4: The behavior of the curve $S(t)$ for the three values k = –1, 0, +1; the time $t = t_0$ is the present time and $t = t_1$ is the time when $S(t)$ reaches zero again for k = +1 .

Let us assume the matter content of the universe as a perfect fluid. The energy momentum tensor $T^{\mu\nu}$ is defined as;

$$T^{\mu\nu} = \rho_0 \, u^\mu u^\nu \tag{10}$$

where ρ_0 is the proper density of matter, and if there is no pressure, and $u^\mu = X^\mu = \frac{dx^\mu}{dt}$ is a tangent vector. A perfect fluid is characterized by pressure $p = p(x^\mu)$, then the energy momentum tensor can be written as;

$$T^{\mu\nu} = (\rho + p)\, u^\mu u^\nu + p g^{\mu\nu}, \tag{11}$$

where ρ is the scalar density of matter.

The principle of local conservation of energy and momentum states that;

$$T^{\mu\nu}_{;\nu} = 0 \cdot \tag{12}$$

Using (11) and (12) in (9) we get;

$$\dot{\rho} + 3(\rho + p)\frac{\dot{S}}{S} = 0\,. \tag{13}$$

Now we solve the Einstein equation for homogeneous and isotropic metric (9) to obtain the following two equations;

$$\frac{3\ddot{S}}{S}+4\pi(\rho+3p)=0 \text{, and} \tag{14}$$

$$\frac{3\dot{S}^2}{S^2}-\left(8\pi\rho-\frac{3k}{S^2}\right)=0 \tag{15}$$

where we have considered $\Lambda = 0$.

For any one of the three values of *k*, we have two equations (14) and (15) for the three unknown functions S, ρ, p. We need one more equation which is provided by the equation of state, $p = p(\rho)$, in which the pressure is given as a function of the mass-energy density. With the equation of state given, the problem is determinate and the three functions S, ρ, p can be worked out complex models of the universe which are determined in this way are reflected to as Friedmann models, after the Russian mathematician A. A. Friedmann (1888–1925) who was the first to study these models.

If $\rho > 0$ and $p \geq 0$ then $\ddot{S} < 0$. So $\dot{S}$ = constant and $\dot{S} > 0$ indicates the universe must be expanding, and $\dot{S} < 0$ indicates contracting universe. The observations by Hubble of the red-shifts of the galaxies were interpreted by him as implying that all of them are receding from us with a velocity proportional to their distances from us that is why the universe is expanding. This could be taken as a verification of the general theory of relativity as coming from the cosmological observations.

For expanding universe $\dot{S} > 0$, so by (14) and (15) we get $\ddot{S} < 0$. Hence $\dot{S}$ is a decreasing function and at earlier times the universe must be expanding at a faster rate as compared to the present rate of expansion. Even if expanded at a constant rate as like the present expansion rate at all times then;

$$\left(\frac{\dot{S}}{S}\right)_{t=t_0} \equiv H_0. \tag{16}$$

Since $\frac{\ddot{S}}{S} < 0$, let the present time be denoted by $t = t_0$. Now $S(t_0) > 0$ and $\frac{\dot{S}(t_0)}{S(t_0)} > 0$ (since we see red shifts, do not see blue shifts); it follows that the curve $S(t)$ must be concave downwards (towards the *t*-axis). It is also clear from the figure–5 that the curve $S(t)$ must reach the *t*-axis at a finite time which is closer to the present time than the time which is the tangent to the point $(t_0, S(t_0))$ reaches the *t*-axis. We refer to the time at which $S(t)$ reaches to the *t*-axis at $t = 0$. Hence at a finite time in the past, namely $t = 0$, we have;

$$S(0) = 0. \tag{17}$$

The point $t = 0$ can reasonably be called the beginning of the universe. Since $\ddot{S} < 0$ for $0 < t < t_0$. Now H_0^{-1} implies a global upper limit for the age of any type of Friedmann models. So the age of the universe will be less than H_0^{-1} i.e.,

$$t_0 < H_0^{-1}. \tag{18}$$

The quantity H_0 is called Hubble constant and at any given epoch it measures the rate of expansion of the universe. By observation H_0 has a value somewhere in the range of 50 to 100 $\text{kms}^{-1}\text{Mpc}^{-1}$. As a result the value of the above upper limit to the age of the universe lies around 10^{10} years with uncertainty of a factor of about two.

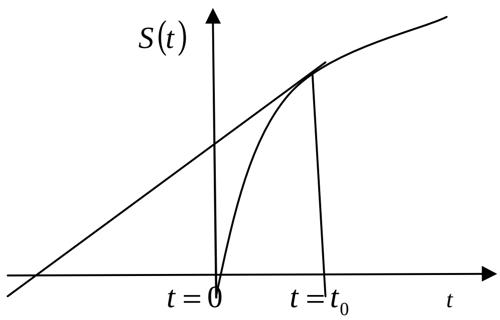

Figure 5: The age of the universe is less than the Hubble time.

At the epoch $S = 0$, the entire three-surface shrinks to zero volume and the densities and curvatures grow to infinity. Hence by FRW models, the universe has an encompassing space-time singularity at a finite time in the past. This curvature singularity is called the big bang (as it were named by Fred Hoyle). Now we have a basic qualitative difference between the Schwarzschild singularity such as that arises at $r = 0$ and that occurring in FRW models [12]. It is clear that both of the singularities occur due to the high degree of symmetry assumed for these models, such as the homogeneity and isotropy assumptions or the spherical symmetry.

During the decades the expansion of the universe was assumed to be decelerating due to gravitation. But the recent measurements of type Ia supernovae (SNe Ia) show that the expansion of the Universe is faster in our epoch than classical models expected [14]. To adjust these observations within the standard cosmology, a repulsive dark energy has been postulated from the concept of cosmological constant.

The Schwarzschild singularity could be the final result of a gravitationally collapsing of massive star. However FRW singularity must be interpreted as the catastrophic event from which the entire universe emerged and where all the known physical laws breakdown in such a way that we cannot know what was there before this singularity. The existence of a strong curvature singularity at $t = 0$ indicated by the FRW models imply the existence of a very hot, dense and radiation dominated region in the very early phase of the evolution of the universe [6, 9].

6 Special Cases in FRW model

At the present epoch in the universe, one could take $p = 0$ as a good approximation when compared with the overall density ρ, then (13) becomes [10];

$$\dot{\rho}+3\rho\frac{\dot{S}}{S}=0$$

$$\frac{d\rho}{dt}+3\frac{\rho}{S}\frac{dS}{dt}=0$$

$$3\frac{dS}{S}=-\frac{d\rho}{\rho}$$

$$3\ln S=-\ln\rho+\ln C$$

$$\ln S^3=\ln\frac{C}{\rho}$$

$$\rho S^3=C \tag{19}$$

which provides the conservation of the rest mass.

On the other hand, for the radiation, such as the microwave background radiation, the equation of state will be $p=\frac{1}{3}\rho$ then (13) becomes;

$$\dot{\rho}+4\rho\frac{\dot{S}}{S}=0$$

$$\frac{d\rho}{\rho}=-4\frac{dS}{S}$$

$$\ln\rho=-4\ln S+\ln C_1$$

$$\rho S^4=C_1. \tag{20}$$

It follows that if the microwave background has a global origin such as the big bang singularity, then its density in the past will grow faster as compared to the matter. So, even though the universe is matter dominated at the moment, it would become radiation dominated in the past, and in the early phases soon after originating from the big bang singularity.

From (15) and (19) for $p = 0$;

$$S^2=\frac{C}{\rho S}$$

$$3\frac{\dot{S}^2}{C/\rho S}-\left(8\pi\rho-\frac{3k}{C/\rho S}\right)=0$$

$$\dot{S}^2=\frac{8\pi C}{3S}-k\,. \tag{21}$$

For $k = 0$ of flat spatial sections which are non-compact and infinite in extent, then (21) becomes;

$$\dot{S}^2=\frac{8\pi C}{3}.\frac{1}{S}$$

$$\frac{dS}{dt}=\sqrt{\frac{8\pi C}{3}}.\frac{1}{\sqrt{S}}$$

$$\frac{S^{3/2}}{3/2}=\sqrt{\frac{8\pi C}{3}}t+C_2 \tag{22}$$

At $t=0, S=0$ then (22) gives $C_2=0$ and using it (22) becomes;

$$S^{3/2}=\sqrt{6\pi C}\,t$$

$$S=(6\pi C)^{1/3}t^{2/3}\,. \tag{23}$$

These spatial sections (and the universe) originated from the big bang singularity the past and then expand forever in time with increasing S. For the case $k=-1$, the spatial sections are hyperboloids of constant negative curvature and they are non-compact and infinite in extent. The solution is given in the parametric form by using (21) as;

$$\dot{S}^2=\left(\frac{dS}{dt}\right)^2=\frac{8\pi C}{3}.\frac{1}{S}+1. \tag{24}$$

The solution is given in the parametric form by;

$$S=\frac{4\pi C}{3}\left(\cosh\eta-1\right), \text{ and} \tag{25}$$

$$t=\frac{4\pi C}{3}\left(\sinh\eta-\eta\right). \tag{26}$$

The universe again originates in the big bang and continues to expand forever in time. In the case $k=+1$ in (21) the solution becomes;

$$\dot{S}^2=\frac{8\pi C}{3}.\frac{1}{S}-1$$

$$S=\frac{4\pi C}{3}\left(1-\cos\eta\right), \text{ and} \tag{27}$$

$$t=\frac{4\pi C}{3}\left(\eta-\sin\eta\right). \tag{28}$$

The spatial sections are now compact three-spheres with constant positive curvature and the radius at time *t* is given by the scale parameter *S*. Here the expansion of the universe from the singularity reaches a maximum and then the scale factor again contracts to zero value ending in a singularity again. The equation for $S(t)$ is a cycloid.

7 Comparison of FRW Models with other Modes

The existence of a strong curvature big bang singularity in the past as indicated by the FRW models imply the existence of a very hot, dense and radiation dominated region in the very early phase of evolution of the universe, near which quantum effects are expected to have played an important role. While a complete quantum gravity description of the big bang is unavailable.

In such a phase, a copious production of elementary particles such as neutrinos could have taken place, which would then expand with the universe. If such particles have a tiny mass, they could constitute a substantial fraction of the total mass-energy density of the universe.

In section–4 we stated the cosmological principle as the requirement that the spacelike hypersurfaces of constant time are homogeneous and isotropic subspaces of the space-time. This leads to the determination of metric as FRW line element for the cosmological space-time and physically it means that there is no preferred position or a preferred direction in the universe for the observer.

The Inflationary model [5] was constructed, using a scalar field whose vacuum energy essentially plays the role of time varying Λ, in order to solve old issues of the big bang such as the horizon, flatness and initial fine-tuning problems [3].

Often, a weaker version and also a strong version of the above cosmological principle are invoked in cosmology, which we discuss below briefly. The weak cosmological principle states that the spacelike surfaces of simultaneity are homogeneous i.e., such surfaces admit three independent spacelike killing vectors at any given point. Physically, for all fundamental observers in a given surface the state of the universe is the same and there is no preferred position in the universe. However, there is no assumption of global isotropy of the matter distribution imposed now. An example of exact solutions of Einstein equations obeying this weak principle is given by the Bianchi cosmological models, which are spatially homogeneous but anisotropic. This would permit rotation and shear in the motion of galaxies which are often considered to be physically important features which one would like to be incorporated in cosmological considerations.

We know that 0.01% of the mass of the stars is converted in radiation during all its luminous life and we know that almost totality of the matter in the universe is composed of stars; therefore we consider that 0.01% of density of matter in the universe is radiation [17].

On the other hand, Bondi and Gold [2] argued for a perfect cosmological principle, which is a stronger version of the cosmological principle and requires that in addition to the homogeneity and isotropy of the spacelike surfaces, the universe must look the same to all fundamental observers at all times as well. It is called the Steady State Theory (SST) of the universe. But the main objection to this model was that it does not preserve the conservation of energy. According to them the universe in expansion just that this does not have a beginning; but as the matter expands, there is loss of density by its expansion (with a velocity according with the Hubble's law). To compensate this loss, Thomas Gold [1] proposed a C field that creates matter with a continuous rate of one atom of hydrogen by cubic meter every 10^{10} years around the entire universe. Their argument was, all the observations in cosmology are made by receiving the light rays which come from the past light cone of the observer, which were emitted a long time ago and hence one must assume the constancy of the laws of physics over this entire time interval. Effectively, their assumption means that there is no preferred position, preferred direction, or any preferred epoch of time in the universe. This amounts to assuming the existence of a timelike Killing vector field in the space-time and this is possible only if the FRW line element either one has $S(t)=\text{constant}$ or $S(t)=\exp(H(t))$ with $H\neq 0$ and the spatial curvature $k=0$. Such cosmological models are called the steady state models and were developed by Bondi and Gold, Hoyle, and Hoyle and Narlikar through different approaches [2, 7, 8]. The line element of this type of model becomes;

$$ds^2=-dt^2+e^{2Ht}\left(dr^2+r^2\left(d\theta^2+\sin^2\theta\, d\phi^2\right)\right). \tag{29}$$

Here *H* could be positive, negative or zero and its value is to be fixed from observations. The value is to be fixed from observations. The value $H = 0$ is ruled out because it leads to a static universe with infinite sky temperature. Similarly, for $H<0$ we have a contracting universe with radiation from distant light sources blue shifted, which again give an infinite sky background. Thus *H* must be positive, corresponding to an expanding universe.

The study of particle creation in the relativistic cosmological models has drawn the attention of a number of authors. The first theoretical approach of the particle creation problem was investigated by Prigogine and Géhéniau [13]. They showed that the second law of thermodynamics may be modified to accommodate flow of energy from the gravitational field to the matter field, resulting in the creation of material particles.

An important difference between the FRW and steady state model is that the later have no singularity of infinite curvature and density either in the past or in the future. This is due to the basic reason that in the steady state theory, the universe is the same as its present state at all times in the past and also in future. Hence the density of the universe is constant at all epochs in a steady state universe. In order to match this with the expansion of the universe the creation of matter at a constant rate is required in the steady state theory. However, considering the present density of the universe which is of the order of 1.67×10^{-30} gmcm^{-3}, and the present expansion rate turns out to be extremely small of the order of 10^{-48} $\mathrm{gmcm}^{3}\mathrm{s}^{-1}$. It would appear that there is no possibility to detect the same with present available instruments.

8 Conclusions

In this study we have tried to give a complete description of FRW models. Some authors considered these models as standard models of the universe. These models open the door to the researchers to form other different types of models. These models are established on the assumptions of homogeneous and isotropy of the universe. If homogeneity or isotropy or both are broken then these models cannot predict about the universe. Actually there is no guarantee that the universe strictly obeys homogeneity and isotropy at all epochs. We have also highlighted some other types of the models of the universe. Yet in the 21st century we could not establish a model of the universe which is acceptable to all, although many have taken attempts to form such a model. Even we do not know the universe is finite or infinite and closed or open. Finally we can say that FRW models support the present observable universe than that of any other model.

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