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Robust Optimization of the Investment Portfolio under Uncertainty Conditions

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Abstract

Of the goal of this study is to investigate the assessment of expected return parameters μ and return covariance matrices Σ in modern portfolio investment tasks. These parameters are used in almost all modern portfolio investment models, including the classic mean-variance Markowitz model, Black-Litterman model, “smart ”models. In practice, they are difficult to evaluate correctly, since the parameter values change every day. However, the quality of investment portfolio depends precisely on these parameters. The quality of investment portfolio is understood as a combination of risk and profitability parameters. Number of methodologies are used in this article to reduce the uncertainty of these parameters. The main idea of these methods is to reduce the sensitivity of resulting optimal portfolios to uncertain input parameters. In other words, if the parameter values μ and Σ change slightly, the final portfolio shall not radically change its structure. According the results gained in this article, one asset will not be able to dominate the final portfolio. Chopra offers using the James-Stein estimate for the expected averages, while Black and Litterman use the Bayesian estimate μ and Σ (taking into account the expert opinions). There are also selection methods and scenarios that are described in detail, for example, in. Of all these methods, the Black-Litterman model is most often used in practice.

Keywords: Black-Litterman model, Portfolio investment, Analysis, Uncertainty, Risk

1 Introduction

Robust optimization principles reduce the impact of the problems described above. To do this, one shall first determine the interval of possible parameter values μ and Σ. The value interval is called an indefinite set of these parameters. The final task will be solved for the "worst" case. As a result, the investor will be able to see the guaranteed level of portfolio income with the “worst ”development of events. Quite often, VaR (Value at Risk) indicator is used as a criterion for the "worst" case (1). Similar approaches are proposed in (2, 3, 4, 5, 6).

2 Text of Article

To solve the problem of constructing optimal portfolios (without taking into account the uncertainty of parameters), the Lagrange method or the Kuhn-Tucker theorem are used (if there are restrictions on the portfolio structure). When solving a robust optimization problem for the “worst ”case from an indefinite set, the use of these methods is inefficient. Instead, the task can be reduced to the class of the second order cone problems (SOCP):

$$Min\{f^T x|\ \|A_i x + b_i\| \leq c_i^T x + d_i,\ i = 1,\dots,N\}. \qquad (1)$$

SOCP is a class of tasks that lies between linear programming (LP) and semi-definite programming (SDP). Quadratic programming problems, problems with hyperbolic constraints, etc. are examples of SOCP class. SOCP can be solved more efficiently than SDP. There are suitable numerical methods for solving SOCP, which are implemented in some software packages. In this work, we used the SEDUMI library - an addition to the MATLAB complex for solving the problems of SOCP and SDP class.

In this work, to give the model the robustness property, the worst case optimization method will be used, and the risk of capital loss will be introduced by defining VaR restrictions and restrictions on the investment portfolio structure. The idea of making models robust by optimizing the worst case is described in (7, 8, 9). To optimize the worst case scenario, one shall first specify the many possible portfolio returns S_m and covariance matrices S_v. This set is called the "indefinite set" in the literature. The scheme for generating indefinite sets for returns and covariances is as follows:

$$\begin{aligned} &\mu_i^L \leq \mu_i \leq \mu_i^U, \quad \forall i \\ &\sigma_{ij}^L \leq \sigma_{ij} \leq \sigma_{ij}^U, \quad \forall i,j \end{aligned} \qquad (2)$$

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$$\mu_i^0 = (\mu_i^L + \mu_i^U)/2,\ \beta_i = (\mu_i^U - \mu_i^L)/2,$$
$$\sigma_{ij}^0 = (\sigma_{ij}^L + \sigma_{ij}^U)/2,\ \delta_{ij} = (\sigma_{ij}^U - \sigma_{ij}^L)/2, \mu_i^0 - \beta_i \le \mu_i \le \mu_i^0 + \beta_i, \quad \forall i$$
$$\sigma_{ij}^0 - \delta_{ij} \le \sigma_{ij} \le \sigma_{ij}^0 + \delta_{ij}, \quad \forall i,j$$

$$S_m = \{\mu: \mu^0 - \beta \le \mu \le \mu^0 + \beta, \beta \ge 0\}$$
$$S_v = \{\Sigma: \Sigma^0 - \Delta \le \Sigma \le \Sigma^0 + \Delta, \Delta \ge 0\}.$$

The formed worst-case optimization problem shall be reduced to SOCP form, after which a robust statement of the original problem will be obtained. Until recently, modern portfolio theory formed by G. Markowitz as far back as 1952 remained almost the only quantitative method for solving the portfolio analysis problem. The main idea of this theory is as follows. Let there be *n* types of assets from which the investor can form a portfolio. Capital is distributed between assets in shares $x_i,\ 0 \le x_i \le 1,\ \sum_{i=1}^n x_i = 1$. Assets are characterized by efficiencies R_i, which are random variables with known mathematical expectations $MR_i=\mu_i$, and covariance matrix $\Sigma = \|cov(R_i, R_j)\|$. Markowitz problem is formulated as follows:

$$\max_x \left\{ \left(\mu^T x - \tfrac{\lambda}{2} x^T \Sigma x\right) \middle| \begin{matrix} I^T x = 1 \\ x \in R_+^n \end{matrix} \right\}. \tag{3}$$

Although the Harry Markowitz model may seem attractive and well-grounded from a theoretical point of view, a number of problems arise in its practical application. Application of the Markowitz model in the Russian market also showed its inconsistency (9). Main disadvantages of the Markowitz model:

- The model does not take into account the fundamental and other factors of profitability;
- The model does not allow for taking into account the uncertainty levels for individual assets;
- With a slight change in the input parameters, one can get a result that is very different from the previous one (instability);
- In the absence of restrictions on the assets structure, there is a large number of negative weights in the final portfolio.

Let us compose a robust model, having previously performed a number of transformations:

$$\begin{aligned} &\underset{x}{Max}\left\{\min_{\mu,\Sigma}\left[\mu^T x - \frac{1}{2}\gamma x^T \Sigma x\right] | I^T x = C_0\right\} \\ &\underset{x}{Max}\left\{\min_{\mu}[\mu^T x] - \frac{1}{2}\gamma \max_{\Sigma}[x^T \Sigma x] | I^T x = C_0\right\} \end{aligned} \tag{4}$$

$$= \sum_i \mu_i^0 x_i + \sum_{i::x_i<0} \beta_i x_i - \sum_{i:x_i \ge 0} \beta_i x_i$$
$$= \sum_i (\mu_i^0 x_i - \beta_i |x_i|)$$
$$= (\mu^0)^T x - \beta^T |x|$$

$$\max_{\Sigma}[x^T \Sigma x] = max \sum_{i,j} \sigma_{ij} x_i x_j$$
$$= \sum_{i,j:x_i x_j<0} (\sigma_{ij}^0 - \delta_{ij}) x_i x_j + \sum_{i,j:x_i x_j \ge 0} (\sigma_{ij}^0 + \delta_{ij}) x_i x_j$$
$$= \sum_{i,j} \sigma_{ij}^0 x_i x_j + \sum_{i,j} \delta_{ij} |x_i x_j|$$
$$= \sum_{i,j} \sigma_{ij}^0 x_i x_j + \sum_{i,j} \delta_{ij} |x_i||x_j|$$
$$= x^T \Sigma^0 x + |x|^T \Delta |x|$$

$$\underset{x}{Max}\left\{(\mu^0)^T x - \beta^T |x| - \tfrac{1}{2}\gamma x^T \Sigma x - \tfrac{1}{2}\gamma |x|^T \Delta |x| | I^T x = C_0\right\}.$$

As a result, the robust task will take the following form:

$$\underset{x,\rho,\tau}{Max}\left\{(\mu^0)^T x - \beta^T |x| - \tfrac{1}{2}\gamma\rho - \tfrac{1}{2}\gamma\tau \middle| \begin{matrix} I^T x = C_0 \\ \rho \ge x^T \Sigma^0 x \\ \tau \ge |x|^T \Delta |x| \end{matrix}\right\}. \tag{5}$$

The Telser model is a logical continuation of the Markowitz model. The main difference and advantage of this model in contrast to the classical statement of the problem of choosing the optimal portfolio is to control the risk of capital loss using the VaR indicator. The model itself has the following formulation:

$$\max_x \left\{ \mu_p \middle| \begin{matrix} VaR_\alpha = -\mu_p - z_\alpha \sigma_p \\ \mu_p = \mu^T x \\ \sigma_p^2 = x^T \Sigma x \\ I^T x = C_0 \\ x \in R_+^n \end{matrix} \right\}. \tag{6}$$

However, this model also inherits the main drawback of the classical approach - a strong instability to the input parameters. We compose a robust model according to the above definitions and prerequisites by performing preliminary transformations:

$$\begin{aligned} &\underset{x}{Max}\left\{\min_{\mu}[\mu^T x] \middle| \begin{matrix} I^T x = C_0 \\ \max_{\mu,\Sigma}[P(R_p \le -VaR_c)] \le \alpha \end{matrix}\right\} \\ &\min_{\mu}[\mu^T x] = (\mu^0)^T x - \beta^T |x| \end{aligned} \tag{7}$$

$$\max_{\mu,\Sigma}[P(R_p \le -VaR_c)] \le \alpha$$
$$\Leftrightarrow \max_{\mu,\Sigma} \frac{-VaR_c - \mu^T x}{\sqrt{x^T \Sigma x}} \le z_\alpha \Leftrightarrow$$
$$\frac{-VaR_c - \min_{\mu} \mu^T x}{\max_{\Sigma} \sqrt{x^T \Sigma x}} \le z_\alpha$$
$$\Leftrightarrow -\min_{\mu} \mu^T x - z_\alpha \max_{\Sigma} \sqrt{x^T \Sigma x} \le VaR_c \Leftrightarrow$$
$$-(\mu^0)^T x + \beta^T |x| - z_\alpha \sqrt{x^T \Sigma^0 x + |x|^T \Delta |x|} \le VaR_c \Leftrightarrow$$
$$-z_\alpha \left\| \begin{pmatrix} \|(\Sigma^0)^{0,5} x\| \\ \|\Delta^{0,5}|x|\| \end{pmatrix} \right\| \le (\mu^0)^T x + \beta^T |x| + VaR_c \ .$$

As a result, the robust task will take the following form:

$$\max_x \left\{ (\mu^0)^T x - \beta^T |x| \;\middle|\; \begin{array}{l} I^T x = C_0 \\ -z_\alpha \left\| \begin{pmatrix} \|(\Sigma^0)^{0,5} x\| \\ \|\Delta^{0,5}|x|\| \end{pmatrix} \right\| \le (\mu^0)^T x + \beta^T |x| + VaR_c \end{array} \right\}. \quad (8)$$

The Black-Litterman model was first published by Fisher Black and Robert Litterman from Goldman Sachs (2,18,19,20). They proposed a theory of "equilibrium approach". Moreover, equilibrium is understood as an idealized state in which demand is equivalent to supply. According to the authors, “natural forces”, the functioning of which eliminates the deviation from equilibrium, function in the economic system. Equilibrium returns are calculated by the formula:

$$\Pi = \lambda \Sigma w_{mkt} \ , \quad (9)$$

where Π - equilibrium return vector; λ - risk aversion coefficient; Σ - covariance matrix of historical returns; w_{mkt} - market capitalization vector of each asset relative to the capitalization amount of assets in the portfolio. The coefficient λ characterizes the investor’s willingness to sacrifice the value of expected portfolio return in order to reduce its risk:

$$\lambda = \frac{E(r) - r_f}{\sigma^2} \ , \quad (10)$$

where $E(r)$ - expected market return, r_f - risk-free interest rate, $\sigma^2 = w_{mkt}^T \Sigma w_{mkt}$ - market portfolio dispersion. Let us consider the Black-Litterman formula for the posterior return vector (7). It is a key point before calculating the final portfolio (6). Let K is the number of subjective opinions, N - the number of assets.

$$\mu = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q] \quad (11)$$

Where μ –new (posterior) mixed return vector $(N \times 1)$; τ –scaling factor; Σ –return covariance matrix with dimension $(N \times N)$; P –dimension matrix $(K \times N)$, which identifies assets for which the investor has a subjective opinion; Ω – diagonal covariance matrix with confidence levels for each subjective opinion, $(K \times K)$; Π –equilibrium return vector, $(N \times 1)$; Q –vector of subjective views, $(K \times 1)$.

Uncertainty of subjective views is reflected in the error vector ε, whose elements are normally distributed with an average of 0 and a matrix Ω. Thus, the final values of subjective opinions have the form of $Q + \varepsilon$.

$$Q + \varepsilon = \begin{bmatrix} Q_1 \\ \vdots \\ Q_k \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_k \end{bmatrix}. \quad (12)$$

Error vector elements ε, usually nonzero. Variations ω of the error vector elements ε form a diagonal covariance matrix Ω and demonstrate the uncertainty measure of subjective views. The matrix is diagonal, because subjective opinions are independent of each other according to the model assumptions.

$$\Omega = \begin{bmatrix} \omega_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \omega_k \end{bmatrix}. \quad (13)$$

There are several methods for determining matrix elements Ω (2, 10).

The values of returns for subjective views, located in the column vector Q, are introduced into the model using the matrix P. The presence of the influence of each subjective opinion is reflected in the line vector of dimension $1 \times N$. Thus, we get the matrix P of dimension $K \times N$ for K views:

$$P = \begin{bmatrix} p_{1,1} & \cdots & p_{1,n} \\ \vdots & \ddots & \vdots \\ p_{k,1} & \cdots & p_{k,n} \end{bmatrix}. \quad (14)$$

The final formula is as follows:

$$w = \mu(\lambda\Sigma)^{-1} \ . \quad (15)$$

Let us make a robust model. In this case, due to the vector formation features for estimating future returns, the use of the previous schemes is unacceptable. To give robustness, we introduce restrictions on the portfolio structure, as well as introduce VaR restrictions to control the risk of capital loss:

$$\max_x \left\{ \mu^T x \;\middle|\; \begin{array}{l} I^T x = 1 \\ AX \le b \\ x^T \Sigma x \le s \\ P\left(Y \le VaR_p(Y)\right) > p \\ Y \in R \\ x \in R_+^n \end{array} \right\}. \quad (16)$$

We preliminary carry out a series of transformations according to the method proposed in (11):

$$P\left(Y \le VaR_p(Y)\right) > p \Leftrightarrow P(\xi^T x \ge -\beta) \ge p \quad (17)$$

$$P(\xi^T x \geq -\beta) = P\left(\frac{\xi^T x - \mu^T x}{x^T \Sigma x} \geq \frac{-\beta - \mu^T x}{\sqrt{x^T \Sigma x}}\right) = 1 - F_{(x)}\left(\frac{-\beta - \mu^T x}{\sqrt{x^T \Sigma x}}\right)$$

$$1 - F_{(x)}\left(\frac{-\beta - \mu^T x}{\sqrt{x^T \Sigma x}}\right) \geq 0.95 \Leftrightarrow F_{(x)}\left(\frac{-\beta - \mu^T x}{\sqrt{x^T \Sigma x}}\right) \leq 0.05$$

$$= \frac{-\beta - \mu^T x}{\sqrt{x^T \Sigma x}} \leq F_{(x)}^{-1}(0.05) = \mu^T x + F_{(x)}^{-1}(0.05)\sqrt{x^T \Sigma x} \geq -\beta \,.$$

As a result, the robust task will take the following form:

$$\max_x \left\{ \mu^T x \left| \begin{array}{l} I^T x = 1 \\ AX \leq b \\ x^T \Sigma x \leq s \\ \mu^T x + F_{(x)}^{-1}(0.05)\sqrt{x^T \Sigma x} \geq -\beta \\ Y \in R \\ x \in R_+^n \end{array} \right. \right\}. \qquad (18)$$

Profitability is one of the most important indicators of portfolio management efficiency, indicating management efficiency. But it is impossible to judge the quality of management strategy using only profitability. In addition to profitability, there is a downside - risk, neglect of it in assessing effectiveness can distort the real state of things. In this work, Sharpe and Schwager coefficients were used to assess the effectiveness of investment portfolio.

In total, several experiments were carried out as part of the work. Time interval: 01.07.2010 –01.02.2011. When conducting experiments on constructing optimal portfolios using the described models, we used data on daily stock quotes traded on the MICEX. The experiments were conducted on the Russian market with ascending and flat trends. Let us first consider the results of experiments with a flat trend. It can be seen that profitability increases for all models, with an increase in risk (Fig. 1, 2). However, robust models have higher returns at approximately the same risk levels. Consequently, the quality of models is increased.

Similarly, let us consider the results of experiments in the Russian market with a ascending trend. It can be seen that profitability increases for all models, with an increase in risk (Fig. 3, 4). However, in case of ascending trend, there is a high return on portfolios both with standard and robust formulations of the problem. The quality of robust models is slightly higher than the quality of models in a standard setting. Again, we can see that the Black-Litterman model dominates, while the classical mean-variance model and the Telser model behave roughly the same. It depends on several reasons.

Similarly, Sharp coefficients were calculated for ascending trend for various risk levels (Fig. 6). Firstly, forecasts from analytical departments with adequate forecasting ability were used (12, 13, 16, 17). Secondly, in the robust formulation of the problem, additional restrictions were introduced on the portfolio structure, which made it possible to maintain the diversification level at higher risks. We draw attention to the behavior of the Sharpe coefficient at various risk levels. Sharp values for lateral trend are shown below (Fig. 5).

3 Methods

In the course of the study, the authors applied the following methods:

1. Selective analysis of specialized literature with a high citation index on the topics indicated in the article title. In particular, we considered the Lagrange method, Kuhn-Tucker theorem, modern portfolio theory of G. Markowitz, Telser model, and Black-Litterman model.

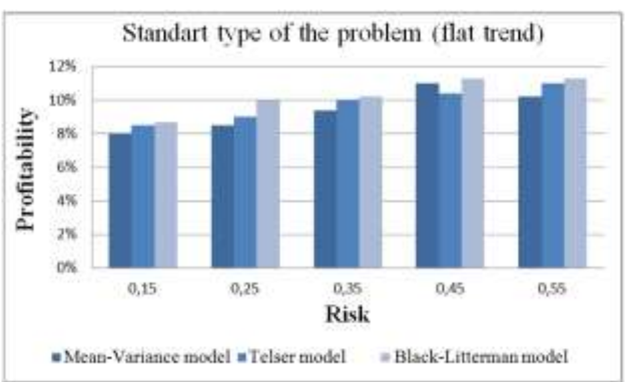

Figure 1: Risks and returns of portfolios with a flat trend

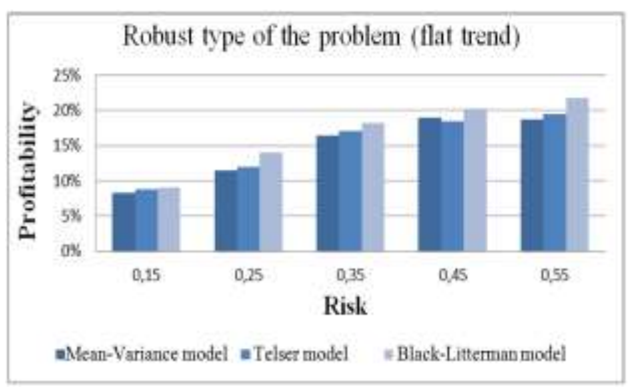

Figure 2: Risks and returns of portfolios with a flat trend

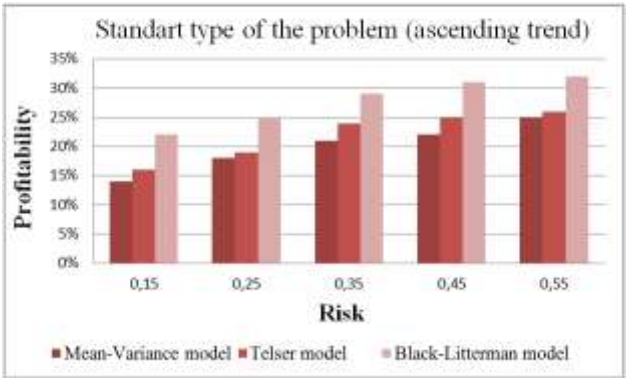

Figure 3: Risks and returns of portfolios with ascending trend

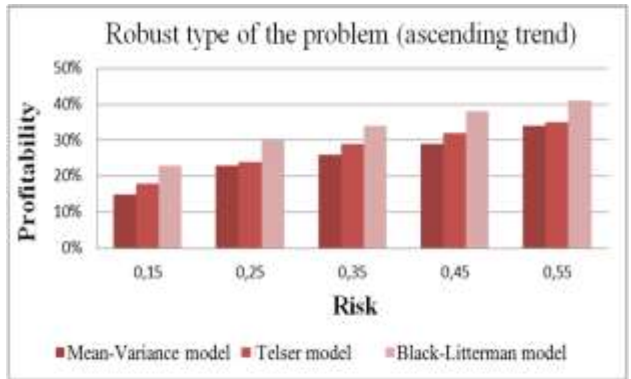

Figure 4: Risks and returns of portfolios with ascending trend

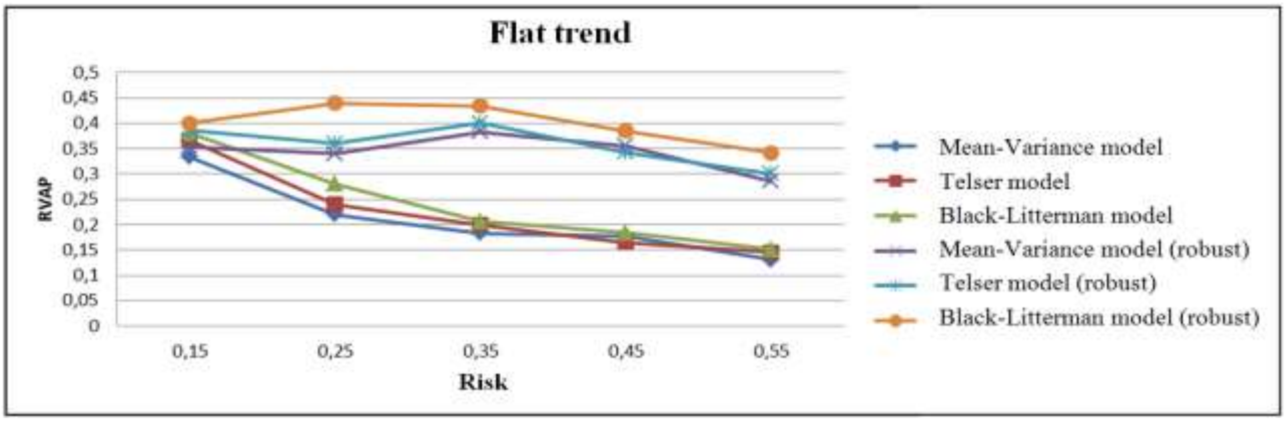

Figure 5: Portfolio quality assessment for flat trend

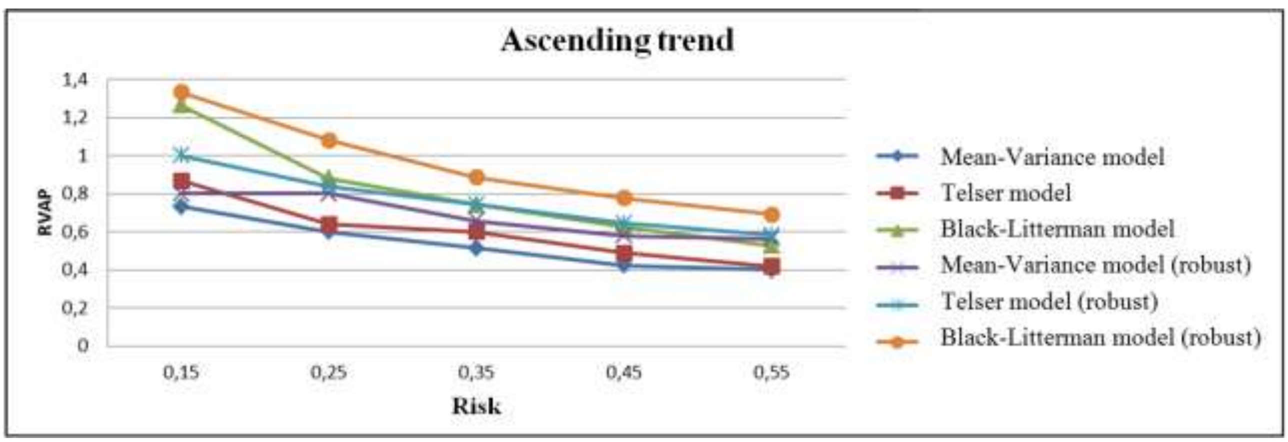

Figure 6: Portfolio quality assessment for ascending trend

2. We carried out a comparative analysis of the collected information according to the criteria defined by the authors in order to identify the advantages and disadvantages of the considered methods and assess the possibility of their practical application.

3. The study results were given the author's interpretation, and we made the respective conclusions.

4 Results and Discussion

According to the results of experiments, one can make the quite expected conclusion that the quality of the investment portfolio (a combination of risk and return indicators) does not depend fundamentally on the portfolio structure with positive ascending trends in the market. At the same time, the portfolio structure plays a decisive role in the uncertainty periods (lateral trend, trend fracture, intervention). In such periods, the portfolio quality will depend on the portfolio structure. These conclusions are typical for models in standard and robust settings. At the same time, robust models have better quality (Fig. 5, 6) compared to the same models in the standard setting in both sections of the trend.

5 Summary

In the framework of the presented study, the following models were subjected to robust optimization: classical mean-variance model, Black-Litterman model, Telser model. We made a comparative analysis of the effectiveness of the models, before robust optimization and after. We evaluated the strengths and weaknesses of different approaches.

6 Conclusions

The expediency of using the robust optimization method is evidenced by the fact that the risk-return ratios for portfolios increased to 5-21% depending on the trend sections, the selected model and the value of selected risk. To assess the quality of investment portfolios, we used coefficients reflecting the risk-free rate, risk and profitability of the portfolios.

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